UNSTEADY TEMPERATURE FIELD IN A SLAB WITH SIMULTANEOUS THERMAL RADIATION AND CONVECTION

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An analysis was made in [1] of an approximate method of calculating the temperature field for the case when heat is transmitted to an object by two simultaneous parallel fluxes; radiative and convective. This method proved to be accurate enough if the heated article is not too massive in the thermal sense, i.e., if the dimensionless parameters Sk and Bi do not exceed 0.5.

The present paper describes a method of determining the temperature field in a slab under the condition that

$$0 \leqslant \mathrm{Sk} < \infty, \quad 0 \leqslant \mathrm{Bi} < \infty.$$

The equation for the unsteady temperature distribution in a plate has the form

$$\frac{\partial \Theta(X, \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2 \Theta(X, \mathrm{Fo})}{\partial X^2} \qquad (0 < X \leqslant 1; \ \mathrm{Fo} > 0). \tag{1}$$

We have to solve this equation under the following boundary and initial conditions:

$$\frac{\partial \Theta(1, \mathrm{Fo})}{\partial X} = \{\mathrm{Bi} + \mathrm{Sk}[1 + \Theta(1, \mathrm{Fo}) + \Theta^2(1, \mathrm{Fo}) + \Theta^3(1, \mathrm{Fo})]\}[1 - \Theta(1, \mathrm{Fo})], \tag{2}$$

$$\frac{\partial \Theta(0, \mathrm{Fo})}{\partial X} = 0, \tag{3}$$

$$\Theta(X, 0) = \Theta_0 = \mathrm{const}. \tag{4}$$

Boundary condition (2) may be written in the somewhat different form

$$\frac{\partial \Theta(1, \mathrm{Fo})}{\partial X} = \mathrm{Bi}^*(\mathrm{Fo})[1 - \Theta(1, \mathrm{Fo})]. \tag{2'}$$

Here the expression Bi*(Fo) is understood to be

$$\mathrm{Bi}^*(\mathrm{Fo}) = \mathrm{Bi} + \mathrm{Sk}[1 + \Theta(1, \mathrm{Fo}) + \Theta^2(1, \mathrm{Fo}) + \Theta^3(1, \mathrm{Fo})]. \tag{5}$$

Thus, the process (1)-(4) may be reduced conditionally to the case of heating with a variable heat transfer coefficient. However, the nature of the dependence of the parameter Bi*(Fo) on time remains as yet unknown in explicit form.

As an approximate analytical solution of Eqs. (1), (2'), (3), and (4) a formula of the following form is valid:

$$\Theta(X, \mathrm{Fo}) = 1 - (1 - \Theta_0)\sum_{n=1}^{\infty} A_n Z_n(\mathrm{Fo}) \times \cos[\mu_n(\mathrm{Fo}) X] \exp - \int_0^{\mathrm{Fo}} \mu_n^2(\mathrm{Fo})\, d\mathrm{Fo}, \tag{6}$$

where $\mu_n(\mathrm{Fo})$ are the roots of the characteristic equation

$$\mu(\mathrm{Fo}) \operatorname{tg}[\mu(\mathrm{Fo})] = \mathrm{Bi}^*(\mathrm{Fo}); \tag{7}$$

A_n are the constant coefficients

$$A_n = \frac{2\sin\mu_n(0)}{\mu_n(0) + \sin\mu_n(0)\cos\mu_n(0)}. \tag{8}$$

The form of the functions

$$Z_n(\mathrm{Fo}) = \left[1 + \frac{\sin^2\mu_n(0)}{\mathrm{Bi}^*(0)}\right]^{1/2}\left[1 + \frac{\sin^2\mu_n(\mathrm{Fo})}{\mathrm{Bi}^*(\mathrm{Fo})}\right]^{-1/2} \tag{9}$$

is established from the condition that

$$\int_0^1 W_n(X, \mathrm{Fo}) \cos[\mu_n(\mathrm{Fo}) X]\, dX = 0,$$

where $W_n(X, \mathrm{Fo})$ is the inviscid solution of (6) and the differential equation (1). Expression (6) rigorously satisfies the boundary conditions (2') and (3), and the initial temperature distribution (4), and is in satisfactory agreement with Eq. (1) for the $Z_n(\mathrm{Fo})$ as determined from (9). In the case Bi* = const, the relation (6) reduces to the exact solution [2].

A first approximation for Bi*(Fo) may be obtained by taking

$$\Theta(1, \mathrm{Fo}) = \Theta_0.$$

Then

$$\mathrm{Bi}_1^* = \mathrm{Bi} + \mathrm{Sk}(1 + \Theta_0 + \Theta_0^2 + \Theta_0^3). \tag{10}$$

From the value of Bi_1^* [and using (6)] a first approximation to $\Theta(X, \mathrm{Fo})$ is found:

$$\Theta_1(X, \mathrm{Fo}) = 1 - (1 - \Theta_0)\sum_{n=1}^{\infty} A_n \cos(\mu_{n1} X) \exp - \mu_{n1}^2 \mathrm{Fo}. \tag{11}$$

Here μ_{n1} are the roots of the equation

$$\mu_1 \operatorname{tg}\mu_1 = \mathrm{B}_1^*.$$

Hence

$$\Theta_1(1, \mathrm{Fo}) = 1 - (1 - \Theta_0)\sum_{n=1}^{\infty} A_n \cos\mu_{n1} \exp - \mu_{n1}^2 \mathrm{Fo}. \tag{11'}$$

The temperature $\Theta_1(1, \mathrm{Fo})$ allows us in turn to find a second approximation for Bi*(Fo):

$$\mathrm{Bi}_2^*(\mathrm{Fo}) = \mathrm{Bi} + \mathrm{Sk}[1 + \Theta_1(1, \mathrm{Fo}) + \Theta_1^2(1, \mathrm{Fo}) + \Theta_1^3(1, \mathrm{Fo})], \tag{12}$$

by means of which, using (6), a new approximation is determined for the temperature field $\Theta(X, \mathrm{Fo})$:

$$\Theta_2(X, \mathrm{Fo}) = 1 - (1 - \Theta_0)\sum_{n=1}^{\infty} A_n Z_{n2}(\mathrm{Fo}) \times \cos[\mu_{n2}(\mathrm{Fo}) X] \exp - \int_0^{\mathrm{Fo}} \mu_{n2}^2(\mathrm{Fo})\, d\mathrm{Fo}. \tag{13}$$

Here $\mu_{n2}(\mathrm{Fo})$ and $Z_{n2}(\mathrm{Fo})$ are calculated from (7) and (9).

The analogous operations may be repeated even further. For practical purposes, however, as a rule, it is sufficient to restrict ourselves to the second approximation $\Theta_2(X, \mathrm{Fo})$.

The table gives the results of calculations for the case when Sk = 0.5, Bi = 1.0, Θ_0 = 0.2. Data from numerical integration of the system (1)-(4), accomplished on an electronic computer [1], are shown, for comparison.

It should be mentioned that the true values of temperature $\Theta(X, \mathrm{Fo})$ turn out to be somewhat larger than the calculated values $\Theta_2(X, \mathrm{Fo})$, since the inequality

$$\mathrm{Bi}^*(\mathrm{Fo}) \geqslant \mathrm{Bi}_2^*(\mathrm{Fo}) \geqslant \mathrm{Bi}_1^*$$

is observed throughout the whole process (the sign = refers only to the time instant when Fo = 0).

If we assume at the first iteration level that

$$\mathrm{Bi}_1^* = \mathrm{Bi} + 4\mathrm{Sk}, \tag{14}$$

Variation of Relative Temperature at the Surface and at the Center of a Plate
($Sk = 0.5$; $Bi = 1.0$; $\Theta_0 = 0.2$)

Fo	Bi_1^*	$\Theta_1(1, Fo)$	$Bi_2^*(Fo)$	$\Theta_2(1, Fo)$	$\Theta(1, Fo)$ from [1]	$\Theta_2(0, Fo)$	$\Theta(0, Fo)$ from [1]
0	1.624	0.2	1.624	0.2	0.2	0.2	0.2
0.1	1.624	0.512	1.954	0.552	0.553	0.201	0.209
0.2	1.624	0.586	2.065	0.636	0.644	0.255	0.262
0.3	1.624	0.633	2.144	0.687	0.699	0.328	0.336
0.4	1.624	0.671	2.212	0.728	0.742	0.401	0.409
0.5	1.624	0.704	2.274	0.763	0.778	0.467	0.478
0.6	1.624	0.733	2.331	0.792	0.808	0.528	0.540
0.8	1.624	0.782	2.436	0.842	0.856	0.632	0.646
1.0	1.624	0.822	2.527	0.880	0.893	0.714	0.729
1.2	1.624	0.855	2.606	0.909	0.919	0.779	0.793
1.6	1.624	0.904	2.729	0.948	0.955	0.870	0.881
2.0	1.624	0.936	2.816	0.971	0.975	0.924	0.932

it is more acceptable for the case of relatively large Sk and Bi that the inverse phenomenon will occur. This is due to the fact that then

$$Bi^*(Fo) \leqslant Bi_2^*(Fo) \leqslant Bi_1^*.$$

The method described may be used for bodies of different geometrical configuration (cylinders, spheres, prisms, etc.), and also for other nonlinear boundary conditions.

NOTATION

$\Theta(X, Fo) = T(X, Fo)/T_m$ is the relative temperature; T_m is the temperature of the medium; T_0 is the initial temperature; δ is the plate half width; α is the heat transfer coefficient; σ_V is the view factor for radiative heat transfer; a is diffusivity; τ is time; $\Theta_0 = T_0/T_m$; $Fo = a\tau/\delta^2$; $Bi = \alpha\delta/\lambda$; $Sk = \sigma_V T_m^3 \delta/\lambda$.

REFERENCES

1. Yu. V. Vidin and V. V. Ivanov, Calculation of Temperature Fields in Solid Bodies Heated by Simultaneous Convection and Radiation [in Russian], Krasnoyarsk Polytechnic Institute, 1965.

2. A. V. Luikov, Theory of Heat Conduction [in Russian], Gostekhizdat, 1952.

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HEAT TRANSFER IN LAMINAR FLOW OF AN INCOMPRESSIBLE FLUID IN A ROUND TUBE

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In the solution of this problem it is usually assumed that heat conduction in the direction of flow is negligibly small in comparison with convective heat transfer. When this assumption is made and when thermophysical characteristics are assumed to be constant and the velocity profile across the tube to be parabolic (which corresponds to steady rectilinear symmetric isothermic laminar flow), the first boundary-value problem can be formulated as follows:

$$\frac{\partial^2 t}{\partial R^2} + \frac{1}{R}\frac{\partial t}{\partial R} = (1 - R^2)\frac{\partial t}{\partial Z},$$

$$0 \leqslant R \leqslant 1, \quad 0 \leqslant Z < +\infty, \tag{1}$$

$$t(0, Z) < +\infty, \tag{2}$$

$$t(1, Z) = f(Z), \tag{3}$$

$$t(R, 0) = \varphi(R). \tag{4}$$

To solve this problem we first solve the corresponding homogeneous problem, viz., Eq. (1) on condition that on the surface of the tube

$$t(1, Z) = 0. \tag{5}$$

We will seek special solutions of this auxiliary problem in the form of products $M(R)\exp(-\mu^2 Z)$ on condition that $M(R)$ is the solution of the following Sturm-Liouville problem [1]:

$$\frac{d^2M}{dR^2} + \frac{1}{R}\frac{dM}{dR} + \mu^2(1 - R^2)M = 0,$$

$$M(0) < +\infty, \quad M(1) = 0. \tag{6}$$

Direct substitution shows that the solution of problem (6) will be a function

$$T(\mu R^2) = F(a, 1, \mu R^2)\exp\left(-\frac{\mu}{2}R^2\right), \tag{7}$$

where $F(a, 1, \mu R^2)$ is a degenerate hypergeometric function, and $a = (2 - \mu)/4$.

Expressing the exponential and hypergeometric function in the form of power series in R^2 [2-4] and multiplying these series, which is possible in view of their absolute convergence, we obtain

$$T(\mu R^2) = 1 + \sum_{k=1}^{\infty}\left(\frac{\mu}{2}\right)^k \times$$

$$\times R^{2k}\sum_{s=0}^{k}(-1)^{s+1}\frac{2^s\Gamma(a+s)}{\Gamma(a)(s!)^2(k-s)!}. \tag{8}$$

To obtain nontrivial solutions we must find the eigenvalues of μ from the equation